Mathematical Analysis - List 3

1. Find the composite functions $f \circ f$, $f \circ g$, $g \circ f$, $g \circ g$ and their domains if

a)
$$f(x) = \frac{1}{x}$$
, $g(x) = x^2$;
b) $f(x) = \sqrt{x}$, $g(x) = x^4$;
c) $f(x) = \frac{1}{x+1}$, $g(x) = \frac{1}{x+2}$;
d) $f(x) = |x|$, $g(x) = \sqrt{x+1}$.

2. Express the function in the form $h = f \circ g$.

a)
$$h(x) = \frac{|x|+1}{|x|-1}$$
; b) $h(x) = \frac{x^2+2x+1}{x^2+2x-1}$; c) $h(x) = \sqrt{\frac{x+1}{x}}$; d) $h(x) = x^4+2x^2-2$.

- **3.** Let f be increasing and g be decreasing on \mathbb{R} . Show that the composite function $f \circ g$ is decreasing on \mathbb{R} .
- **4.** How is the graph of y = f(|x|) related to the graph of y = f(x)?
- 5. Find a formula for the inverse of the function.

a)
$$f(x) = \frac{1+3x}{5-2x}$$
;
b) $f(x) = \frac{1+3^x}{1-3^x}$;
c) $f(x) = 5-4x^3$;
d) $f(x) = \log_3(x+2)$.

6. If
$$f(x) = 4 + x^3 + \tan\left(\frac{\pi x}{2}\right)$$
, where $-1 < x < 1$, find
a) $f^{-1}(4)$; b) $f^{-1}\left(5\frac{1}{8}\right)$; c) $f^{-1}\left(2\frac{7}{8}\right)$.

7. Find the exact value of each expression.

- a) $\log_6 \frac{1}{36}$; b) $\log_{10} 1.25 + \log_{10} 80$;
- c) $\log_5 10 + \log_5 20 3\log_5 2;$ d) $\log_2 2^{\sqrt{3}}.$
- 8. Starting with the graph of $y = \log_2 x$, find the equation of the graph that results from
 - a) reflecting about the x-axis,
 - b) reflecting about the line y = x,
 - c) reflecting about the *y*-axis and then about the line y = x,
 - d) shifting 3 units to the left and then reflecting about the line y = x.
- 9. Find the domain and range of the function

$$g(x) = \log_3(9 - x^2).$$

10. Solve each equation for x.

a)
$$\ln(2x - 1) = 3;$$

b) $2^{x-5} = 3;$
c) $\ln(\ln x) = 1;$
d) $\ln x + \ln(x - 1) = 1$