## Mathematical Analysis - List 3

1. Find the composite functions $f \circ f, f \circ g, g \circ f, g \circ g$ and their domains if
a) $f(x)=\frac{1}{x}, g(x)=x^{2}$;
b) $f(x)=\sqrt{x}, g(x)=x^{4}$;
c) $f(x)=\frac{1}{x+1}, g(x)=\frac{1}{x+2}$;
d) $f(x)=|x|, \quad g(x)=\sqrt{x+1}$.
2. Express the function in the form $h=f \circ g$.
а) $h(x)=\frac{|x|+1}{|x|-1}$;
b) $h(x)=\frac{x^{2}+2 x+1}{x^{2}+2 x-1}$;
c) $h(x)=\sqrt{\frac{x+1}{x}}$;
d) $h(x)=x^{4}+2 x^{2}-2$.
3. Let $f$ be increasing and $g$ be decreasing on $\mathbb{R}$. Show that the composite function $f \circ g$ is decreasing on $\mathbb{R}$.
4. How is the graph of $y=f(|x|)$ related to the graph of $y=f(x)$ ?
5. Find a formula for the inverse of the function.
a) $f(x)=\frac{1+3 x}{5-2 x}$;
b) $f(x)=\frac{1+3^{x}}{1-3^{x}}$;
c) $f(x)=5-4 x^{3}$;
d) $f(x)=\log _{3}(x+2)$.
6. If $f(x)=4+x^{3}+\tan \left(\frac{\pi x}{2}\right)$, where $-1<x<1$, find
a) $f^{-1}(4)$;
b) $f^{-1}\left(5 \frac{1}{8}\right)$;
c) $f^{-1}\left(2 \frac{7}{8}\right)$.
7. Find the exact value of each expression.
a) $\log _{6} \frac{1}{36}$;
b) $\log _{10} 1.25+\log _{10} 80$;
c) $\log _{5} 10+\log _{5} 20-3 \log _{5} 2$;
d) $\log _{2} 2^{\sqrt{3}}$.
8. Starting with the graph of $y=\log _{2} x$, find the equation of the graph that results from
a) reflecting about the $x$-axis,
b) reflecting about the line $y=x$,
c) reflecting about the $y$-axis and then about the line $y=x$,
d) shifting 3 units to the left and then reflecting about the line $y=x$.
9. Find the domain and range of the function

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g(x)=\log _{3}\left(9-x^{2}\right) .
$$

10. Solve each equation for $x$.
a) $\ln (2 x-1)=3$;
b) $2^{x-5}=3$;
c) $\ln (\ln x)=1$;
d) $\ln x+\ln (x-1)=1$.
